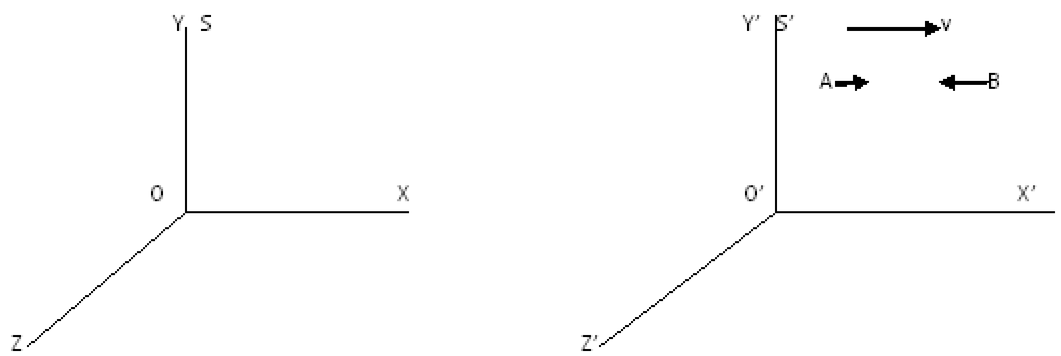
Variation of mass with velocity and its derivation

There is variation of mass with velocity in relativity that is mass varies with the velocity when the velocity is comparable with the velocity of the light. Let us derive and discuss the variation of mass with the velocity relation:

Let there are two inertial frames of references S and S1. S is the stationary frame of reference and S' is the moving frame of reference. At time t=t'=0 that is in the start, they are at the same position that is Observers O and O' coincides. After that S' frame starts moving with a uniform velocity v along x axis.

Suppose there are two particles moving in opposite direction in frame S'. velocity of particle A will be u' and of B will be –u' according to the observer O'.
Let us study the velocities and mass of these particles from frame S.



Velocity of A is u1 and B is u2 from frame S and these are given

by relativistic addition of velocity relation respectively:

$u1 = (u' + v)/(1 + u'v/c^2)$ (1)
$u2 = (-u' + v)/(1 - u'v/c^2)$ (2)

Let m1 and m2 are the mass of A and B from frame S respectively.

As the particles are moving to each other, at certain instant they will collide and momentarily came to rest. But even when they came to rest, they travel with the velocity of the frame S' that is with v.

According to the law of conservation of momentum:

Momentum before collision = momentum after collision

Thus $m1u1 + m2u2 = (m1 + m2)v = m1v + m2v$

Or $m1(u1 – v) = m2(u2 – v)$

Put equations (1) and (2) in above equations, we get

$m1[(u' + v)/(1 +u'v/c^2) – v] = m2[v – (-u' + v)/(1 -u'v/c^2)]$

Then take LCM of terms in the bracket and solve, we get

$m1[1/(1 +u'v/c^2)] = m2[1/(1 -u'v/c^2)]$

or $m1/m2 = (1 +u'v/c^2)/ (1 -u'v/c^2)$ (3)

Now square equation (1), then divide both sides by c^2 and subtract both sides by 1, we get

$1 – u1^2/c^2 = 1 – [(u' + v)/c/(1 + u'v/c^2)]^2$

By taking LCM on RHS and solving, we get

$1 – u1^2/c^2 = (1 + u'^2v^2/c^4 – u'^2/c^2 – v^2/c^2)/ (1 + u'v/c^2)^2$ (4)

Similarly by squaring equation (2), then dividing both sides by c^2 and

subtracting both sides by 1, we get

$1 – u2^2/c^2 = (1 + u'^2v^2/c^4 – u'^2/c^2 – v^2/c^2)/ (1 - u'v/c^2)^2$ (5)

On dividing equation (5) by (4), we get

$(1 – u2^2/c^2)/(1 – u1^2/c^2) = (1 + u'v/c^2)^2/(1 - u'v/c^2)^2$

Take square root on both sides

$(1 – u2^2/c^2)^{1/2}/(1 – u1^2/c^2)^{1/2} = (1 + u'v/c^2)/(1 - u'v/c^2)$ (6)

Now compare equations (3) and (6), we get

$m1/m2 = (1 – u2^2/c^2)^{1/2}/(1 – u1^2/c^2)^{1/2}$ (7)

This is more of a complicated result. To make this result simple, let us assume

that the particle B is in the state of rest from frame S that is it has zero velocity

before collision

Thus $u2 = 0$

And $m2 = m0$

Where m0 is the rest mass of the particle, Therefore equation (7) becomes

$$m1/m0 = 1 /(1 - u1^2/c^2)^{1/2}$$

Also assume u1 = v and m1 = m

Therefore above equation becomes

$$m/m0 = 1 /(1 - v^2/c^2)^{1/2}$$

$$\text{or } m = mo /(1 - v^2/c^2)^{1/2} \qquad (8)$$

This equation represents the equation of the variation of mass with the velocity.